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Separable Coordinate Systems for PDEs

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Under ONR contract N00014-87-K-0191 we studied the following problems:

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Extension of Separation of Variables

Using the symmetry groups of the equations governing acoustic and electromagnetic scattering (the Helmholtz and vector Helmholtz equations, respectively) we studied new methods for separating the equations into ordinary differential equations and generating new classes of solutions in novel geometries.

There are many methods for generating solutions to partial differential equations, including finite elements and finite difference numerical techniques. The oldest method, and one of the most powerful, is the method of separation of variables, invented by Bernoulli. We define "separation of variables" to mean the decomposition of a partial differential equation into a set of uncoupled ordinary differential equations. This is useful, as the solution of ordinary differential equations by numerical techniques is much easier and faster than the solution of partial differential equations.

This work is ~~very challenging and~~ long-term in nature, and though this project has not yet been completed, progress under the contract was ~~encouraging and offers~~ hope that eventually this approach will be successful. We attacked two specific problems with these methods:

(1) electromagnetic scattering from paraboloidal surfaces; AND

Conventional Helmholtz and Hertz decomposition techniques fail to produce analytic solutions to this problem, and the only means of investigating the solutions involves asymptotic limits and numerical techniques. We produced an iterative method based on a two-term recursion relation (similar to the ladder operator method for the quantum mechanical harmonic oscillator) for generating the solutions of the vector Helmholtz equation in this geometry. The "ground state" solution is found, and then higher order solutions are generated that satisfy boundary conditions along paraboloidal surfaces. These solutions not only shed light on the geometric nature of the solutions of vector partial differential equations, but allow one to solve electromagnetic propagation problems (e.g., radar scattering calculations) more accurately. We produced a family of new solutions, and a manuscript is in preparation.

(2) acoustic wave interaction with unusual shapes. (KR)

We found a class of new geometries that are candidate coordinate systems for separation of the scalar Helmholtz equation (i.e., the Fourier transformed acoustic wave equation). These new curves are significantly different from the conventional quadratic curves formed by the standard coordinate systems in which the acoustic equation separates. We did not manage to extract new solutions in any of these geometries, but with continued effort, we anticipate success.

Singular Perturbation Methods at the Sea Floor

We analyzed the problem of acoustic and elastic wave propagation at "soft" fluid-solid boundaries such as the sea floor, discovering that in some circumstances, the wave propagation problem can be accurately solved using conventional methods such as generalized ray theory or WKJB methods. To obtain complete solutions, the system must be studied using singular perturbation methods. We cast the full three-dimensional problem in a set of coupled ordinary differential equations in the fluid, elastic solid and soft solid regions, including the effects of compressibility and viscosity suitable for perturbation analysis leading to accurate numerical solutions.

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